Art of Problem Solving Textbooks

Are You Ready For

Introduction to Algebra

This diagnostic test consists of two parts, **Fundamentals** and **Problem Solving**. If your student can solve nearly all of the **Fundamentals** problems and at least half of the **Problem Solving** problems, then the student is ready for the Art of Problem Solving textbook **Introduction to Algebra**.

If the student cannot solve more than 80% of the **Fundamentals** section, then the student should consider our **Prealgebra** text. If the student solves nearly all of the **Fundamentals** but has a great deal of difficulty with the **Problem Solving** problems, then the student should consider reviewing our **Prealgebra** text, which will help the student develop the maturity and problem-solving skills needed for our **Introduction to Algebra** text.

We recommend using the following process in administering this diagnostic:

Step 1: The student should attempt all of the questions below without a calculator and without any help. There is no time limit.

Step 2: Check the student's responses using the answer key at the end of this document.

Step 3: The student should be given a second chance on the problems that he or she answered incorrectly.

Fundamentals

1. **Working with Variables**.
   (a) Expand the product $9(3x + 7)$.
   (b) Simplify $7a - 5b + 3(6a + b)$.
   (c) Simplify $a - (-7a - 3)$.
   (d) Simplify $3(5 - 2r) - 2(-3r + 1)$.
2. **Fractions, Decimals, and Basic Percents**.
   (a) Convert 0.6144 to a fraction in simplest form.
   (b) Convert 17/40 to a decimal.
   (c) 17 is what percent of 20?
   (d) What is $\frac{9}{5} - \frac{4}{3}$?
   (e) What is $\frac{3/6}{2/5}$ in simplest form?

3. **Linear Equations**. Solve each of the following equations:

   (a) $3r - 4 = 16 - 7r$ (b) $\frac{2x-3}{5} = \frac{4-3x}{7}$ (c) $2 - \frac{t}{4} = 3\left(5 - \frac{t}{6}\right)$

4. **Exponent Laws**. Express each of the following as a power of 2:

   (a) $2^7 \cdot 2^6$ (b) $\frac{2^{13}}{2^5}$ (c) $(2^4)^3$ (d) $2^3 \cdot \left(\frac{2^7}{2^2}\right)^3$

5. **Ratio and Rates**.

   (a) The ratio of boys to girls at a summer camp is 4 to 5. If the total number of students at the camp is 108, then how many boys are at the camp?

   (b) The ratio of teachers to students in a particular school is 1 to 11. The ratio of female students to the total number of students is 4 to 9. If there are 396 female students, then how many teachers are there?

   (c) A train is traveling 1 mile every 75 seconds. If the train continues at this rate, then how far will it travel in two hours?

6. **Square Roots**. Simplify each of the following as much as possible:

   (a) $\sqrt{81}$ (b) $\sqrt{144}$ (c) $\sqrt{1\frac{7}{9}}$ (d) $\frac{\sqrt{108}}{\sqrt{3}}$

Problem Solving

7. What is the value of the sum $5 + 10 + 15 + \cdots + 95 + 100$?

8. Two-fifths of the students in Central Middle School are boys. One-third of the girls have blond hair and one-quarter of the boys have blond hair. What fraction of the students in Central Middle School have blond hair?

9. Kayla adds the same number to both the numerator and denominator of the fraction $\frac{1}{10}$. Her resulting fraction equals $\frac{2}{3}$. What number did she add to both the numerator and denominator of her original fraction?

10. Five workers together can build a road in 20 days. Suppose every worker works at the same rate. Three workers work on the road for 10 days before eleven more workers join them. How much longer will it take the fourteen workers to finish the road?

11. In rectangle $ABCD$, point X is the midpoint of $\overline{AD}$ and Y is the midpoint of $\overline{CD}$. What fraction of the area of the rectangle is enclosed by $\triangle AXY$?

12. At Annville Junior High School, 30% of the students in the Math Club are in the Science Club, and 80% of the students in the Science Club are in the Math Club. There are 15 students in the Science Club. How many students are in the Math Club?

The answers to Are You Ready for Algebra 1 are below.

1. (a) $27x + 63$; (b) $25a - 2b$; (c) $8a + 3$; (d) 13
2. (a) 384/625; (b) 0.425; (c) 85%; (d) 7/15; (e) 5/4
3. (a) $r = 2$; (b) $x = \frac{41}{29}$; (c) $t = 52$
4. (a) 2^{13}; (b) 2^8; (c) 2^{12}; (d) 2^{18}
5. (a) 48 boys; (b) 81 teachers; (c) 96 miles
6. (a) 9; (b) 12; (c) $\frac{4}{3}$ or $1\frac{1}{3}$; (d) 6
7. 1050
8. $\frac{3}{10}$
9. 17
10. 5 days
11. $\frac{1}{8}$
12. 40